УДК 533.6

DOI: 10.18384/2310-7251-2021-4-75-85

ТУРБУЛЕНТНАЯ СТАТИСТИКА С ТОЧКИ ЗРЕНИЯ КОГЕРЕНТНОЙ СТРУКТУРЫ В ПОГРАНИЧНОМ СЛОЕ

Селим Р. С.^{1,2}

¹ *Московский физико-технический институт (национальный исследовательский университет)*

141701, г. Долгопрудный, Московская область, Институтский пер., д. 9, Российская Федерация

² *Университет Танта, факультет естественных наук, математический факультет Танта 31512, улица Аль-Гейш, г. Танта, Египет*

Аннотация

Цель работы: исследование эргодичности динамической системы уравнений для амплитуд набора волн Толлмина–Шлихтинга, с помощью которых аппроксимируется стохастическое поведение турбулентных пульсаций, и проведение сравнения с экспериментом пульсационных характеристик развитого турбулентного пограничного слоя при нулевом градиенте давления в рамках волноводной модели.

Процедура и методы. В работе использована машинная аналитика и комплекс прикладных программ MATHEMATICA. Ряд результатов получен на основе теории динамических систем.

Результаты. Показана сходимость временных средних и средних по пространству. Кроме того, продемонстрирована хорошая сходимость теоретических результатов, обусловленных когерентной составляющей, с экспериментальными.

Теоретическая и практическая значимость. Показана применимость волноводной модели для изучения статистики пульсаций скорости потока в пограничном слое для несжимаемой жидкости при нулевом градиенте давления.

Ключевые слова: вязкая несжимаемая жидкость, когерентные структуры, турбулентный пограничный слой, волноводная модель.

Благодарность. Исследователь Р. С. Селим финансируется за счет частичной стипендии Министерства высшего образования Арабской Республики Египет и стипендии Правительства Российской Федерации с приложением №. EGY-6177/17.

TURBULENT STATISTICS IN TERMS OF COHERENT STRUCTURE IN THE BOUNDARY LAYER

R. S.Selim^{1,2}

¹ *Moscow Institute of Physics and Technology (National Research University)*

9 Institutskypereulok, Dolgoprudny 141701, Moscow region, Russian Federation

² *Tanta University, Faculty of Science, Mathematics Department, Al-Geish Street, Tanta 31512, Egypt*

Abstract

Aim of study: investigation the ergodicity of a dynamical system for a set equations of amplitudes Tollmien-Schlichting waves, with the help of approximation stochastic behavior of turbulent pulsations, and comparison the pulsation characteristics of a developed turbulent boundary layer in terms of the waveguide model with experiment at zero pressure gradient.

Methodology. The paper uses machine analytics and a complex of MATHEMATICA application programs. A number of results are obtained on the basis of dynamical systems theory.

Results. The convergence of time averages and phase averages is shown. In addition, a good convergence of the theoretical results due to the coherent component with the experimental results has been demonstrated.

Research implication. The applicability of the waveguide model for studying the statistics of flow velocity pulsations in the turbulent boundary layer for an incompressible fluid at zero pressure gradient is shown.

Keywords: Viscous incompressible fluid, coherent structures, turbulent boundary layer, waveguide model.

Acknowledgments. The researcher R. S. Selim is funded by a partial scholarship from the Ministry of Higher Education of the Arab Republic of Egypt, and by scholarship from Russian government scholarship with application no. EGY – 6177/17.

1. Introduction

In [1] Navier-Stokes equations is formulated to obtain a waveguide model [2-4] that is stochastically closed relative to the pulsation characteristics of a turbulent boundary layer. This model contains a small parameter that allowed using the method of multiple scales (the method of singular perturbations). According to the triple decomposition [5], the velocity field is divided into three components: average, coherent and stochastic. As a result, equations for the coherent component are obtained [1]:

$$\frac{\partial \bar{U}}{\partial T} + \bar{U} \frac{\partial \bar{U}}{\partial X} + \bar{V} \frac{\partial \bar{U}}{\partial y} = \frac{\partial}{\partial y} \left(-\overline{u'v'} + \frac{1}{\varepsilon^2 R} \frac{\partial \bar{U}}{\partial y} \right). \quad (1)$$

$$\frac{da_1}{dt_1} = \sum_{s=1}^n \Lambda_1^s a_2^s a_3^s, \quad \frac{da_2^s}{dt_1} = \Lambda_2^s a_1 a_3^{s*}, \quad \frac{da_3^s}{dt_1} = \Lambda_3^s a_1 a_2^{s*}. \quad (2)$$

Solutions of the dynamical system (2) obey the invariant

$$I^{(n)}(t_1) = |\tilde{a}_1|^2 + \sum_s (q_{12}^s |\tilde{a}_2^s|^2 + q_{13}^s |\tilde{a}_3^s|^2) = \text{const}. \quad (3)$$

Then, after using the following transformation

$$b_2^s = \sqrt{q_{12}^s} a_j^s, \quad j = 2, 3, \quad a_i(t_1, t_2) = \varphi(t_2) \tilde{a}_i(\varphi(t_2) \tilde{t}_1, t_2). \quad (4)$$

It can be represented as:

$$I^{(n)}(t_1) = |b_1|^2 + \sum_s (|b_2^s|^2 + |b_3^s|^2) = 1, \quad s = 1, 2, \dots, n. \quad (5)$$

Here $(\bar{U}, \bar{V})$ – average longitudinal and transverse velocity components, u_∞ – the velocity of free stream flow, ν – kinematic viscosity, ε – small parameter, $\Lambda_j^s, j = 1, 2, 3, s = 1, 2, \dots, n$ coefficients determined by integral characteristics of the eigenfunctions of the Orr-Sommerfeld equation [1], a_1, a_j^s – amplitudes of Tollmien-Schlichting waves in the state of multiple 3-wave resonance, $I^{(n)}(t_1)$ – invariant [6] of system equations (2), q_{12}^s, q_{12}^s are determined through coefficients Λ_1^s, Λ_2^s и $\Lambda_3^s, s = 1, 2, \dots, n$, n – the number of subharmonics considered, φ – coefficient in equation (4) represents proportionality in the law of similarity [2], и $\overline{u'v'}$ is defined as the time average of the velocity components:

$$\overline{u'v'} = \lim_{t_1 \rightarrow \infty} \frac{1}{t_1} \int_0^{t_1} \left(\lim_{t_0 \rightarrow \infty} \frac{1}{t_0} \int_0^{t_0} \dot{u} \dot{v} dt_0 \right) dt_1, \quad (6)$$

$$\tau_0 = \frac{tu_\infty}{d}, \quad \tau_1 = \varepsilon \tau_0, \quad \varepsilon^2 = \frac{d}{L}.$$

On the basis of the waveguide model, the coefficients Λ_1^s, Λ_2^s and Λ_3^s in equation (2) describing the dynamics of Tollmien-Schlichting waves and their dynamics determine the unsteady behaviour of turbulent pulsations in the boundary layer. In this paper, the average characteristics of pulsations are determined by averaging non-stationary solutions over time. In addition to, the Birkhoff-Khinchin theorem [7] in the representation of A.N. Kolmogorov is applied, due to normalization of invariant to unity. The structure of the dynamical system corresponds to the Birkhoff – Khinchin theorem allowed us to ensure that the dynamical system is ergodic, since the time average is equal to the phase average, on the surface of unit sphere. As a result, the elements of the pulsation tensor are calculated and compared with numerical and experimental data.

2. Statement problem

Using the Fourier representation of the continuity equation and the normal vorticity component, which are expressed by:

$$u'_{\omega k} = \frac{i}{k^2} \left(\alpha \frac{dv'_{\omega k}}{dy} - \beta \eta'_{\omega k} \right),$$

$$(-\omega(\mathbf{k}) + \alpha \bar{U}) \eta'_{\omega k} + \beta \frac{d\bar{U}}{dy} v'_{\omega k} = 0,$$

Also an expression for the vertical component of the velocity is formulated as:

$$v'_{\omega k}(t_0, t_1) = A_{\omega k}(t_1) \delta(\omega - \omega_{T-S}(k)) e^{i\omega t_0 - ik \cdot r} \varphi_{T-S}^{(0)}(\mathbf{k}, y),$$

$$A = A^c + A', \quad \langle A \rangle = A^c, \quad \langle A' \rangle = 0,$$

$$A_{\omega k}(t_1) = a_1(t_1)\delta(\mathbf{k} - \mathbf{k}_1) + a_1^*(t_1)\delta(\mathbf{k} + \mathbf{k}_1) + \\ + \sum_s a_2^s(t_1)\delta(\mathbf{k} - \mathbf{k}_2^s) + a_2^{s*}(t_1)\delta(\mathbf{k} + \mathbf{k}_2^s) + \\ + \sum_s a_3^s(t_1)\delta(\mathbf{k} - \mathbf{k}_3^s) + a_3^{s*}(t_1)\delta(\mathbf{k} + \mathbf{k}_3^s),$$

where $\varphi^{(0)}$ is the least damping mode eigenfunction, $A_{\omega k}(t_1)$ is the amplitude Tollmien-Schlichting, and A^c, A^s are the coherent and stochastic parts respectively. Here, for simplicity, we dropped the mark ($'$) of pulsation variables to avoid the confuse with the derivative to the normal component y . By determining the time average (6), we obtain the distribution of pulsations of the longitudinal velocity:

$$\bar{u}^2 = \frac{\varphi^2}{(2\pi)^6} \left(g_1(\mathbf{k}_1, y) + 2g_2(\mathbf{k}_1^s, y) \frac{dU}{dy} + 2g_3(\mathbf{k}_1^s, y) \left(\frac{dU}{dy} \right)^2 \right), \quad (7)$$

$$g_1(\mathbf{k}_1, y) = -\frac{|a_1|^2 \alpha_1^2}{|k_1|^4} |\dot{\varphi}'_1(\mathbf{k}_1, y)|^2 - 2 \sum_{s=1}^n \sum_{l=2}^3 \frac{|a_l^s|^2}{q_{1,l}^s} \alpha_l^{s2} \frac{|\dot{\varphi}'_l(\mathbf{k}_l^s, y)|^2}{|k_l^s|^4},$$

$$g_2(\mathbf{k}_1^s, y) = -\frac{|a_1|^2 \alpha_1 \beta_1^2}{|k_1|^4} \left(\frac{\dot{\varphi}'_1^*(\mathbf{k}_1, y) \varphi_1(\mathbf{k}_1, y)}{(\omega_r(\mathbf{k}_1) - \alpha_1 U(y) + i\omega_i(\mathbf{k}_1))} + \frac{\dot{\varphi}'_1(\mathbf{k}_1, y) \varphi_1^*(\mathbf{k}_1, y)}{(\omega_r(\mathbf{k}_1) - \alpha_1 U(y) - i\omega_i(\mathbf{k}_1))} \right) -$$

$$- \sum_{s=1}^n \sum_{l=2}^3 \frac{|a_l^s|^2}{q_{1,l}^s} \alpha_l^s \beta_l^{s2} \left(\frac{\dot{\varphi}'_l^*(\mathbf{k}_l^s, y) \varphi_l(\mathbf{k}_l^s, y)}{(\omega_r(\mathbf{k}_l^s) - a_l^s U(y) + i\omega_i(\mathbf{k}_l^s))} + \frac{\dot{\varphi}'_l(\mathbf{k}_l^s, y) \varphi_l^*(\mathbf{k}_l^s, y)}{(\omega_r(\mathbf{k}_l^s) - a_l^s U(y) - i\omega_i(\mathbf{k}_l^s))} \right),$$

$$g_3(\mathbf{k}_1^s, y) = -\frac{|a_1|^2 \beta_1^4}{|k_1|^4 \left((\omega_r(\mathbf{k}_1) - \alpha_1 U(y))^2 + \omega_i(\mathbf{k}_1)^2 \right)} |\varphi_1(\mathbf{k}_1, y)|^2 - \\ - \sum_{s=1}^n \sum_{l=2}^3 \frac{|a_l^s|^2}{q_{1,l}^s} \beta_l^{s4} \frac{|\varphi_l(\mathbf{k}_l^s, y)|^2}{|k_l^s|^4 \left((\omega_r(\mathbf{k}_l^s) - a_l^s U(y))^2 + \omega_i^2(\mathbf{k}_l^s) \right)},$$

where $\varphi'_1 = \frac{d\varphi_1}{dy}$, U – average longitudinal velocity [8], $q_{12}^{(j)}(\beta)$ – represent the weight

factors in formula (5), as displayed in Fig. 1 [6], $\langle |a_j^s|^2 \rangle$, $j = 1, 2, 3$, $s = 1, 2, \dots, n$ – the phase average values of the squares harmonics and subharmonics amplitudes, that defined on the surface of the unit sphere.

Fig. 1. Behavior of multipliers $q_{12}^{(j)}(\beta)$ depending on the transverse component of the wave vector [6].

In addition, (ω_r, ω_i) – are the real and imaginary parts of the least damping mode, which are obtained from the solution of the spectral Orr-Sommerfeld problem [9] on the profile of the average longitudinal velocity in the developed turbulent boundary layer [8]. Normalization of invariant (5), to unity, is contributed to apply Birkhoff-Khinchin theorem [7]. As a result, a good agreement is noticed between the time, and phase averages of the dynamical system (2). Thus, the real and imaginary part of the amplitude (b) in equation (5) are further written in terms of the spherical coordinates x_i , $i = 1, 2, \dots, n$, in n -dimensional space [10]:

$$I = |x_1|^2 + |x_2|^2 + \dots + |x_{41}|^2 + |x_{42}|^2 = 1,$$

where

$$x_1 = \sin \theta_1 \sin \theta_2 \dots \sin \theta_{n-1},$$

$$x_m = \cos \theta_{m-1} \prod_{k=m}^{n-1} \sin \theta_k, \quad m = 2, 3, \dots, n-1,$$

$$x_n = \cos \theta_{n-1},$$

where $\theta_1, \theta_2, \theta_3, \dots, \theta_{n-1}$ are the spherical angles, that changed as $0 \leq \theta_1 < 2\pi$, $0 \leq \theta_m \leq \pi$ at $m = 1, 2, \dots, n-1$, on the surface of the sphere S_n :

$$\begin{aligned} S_n &= \int_{\theta=0}^{2\pi} \int_{\varphi_1=0}^{\pi} \int_{\varphi_2=0}^{\pi} \dots \int_{\varphi_{n-2}=0}^{\pi} J_n d\theta d\varphi_1 d\varphi_2 \dots d\varphi_{n-2} = \\ &= \int_{\theta=0}^{2\pi} d\theta \prod_{k=1}^{n-2} \int_{\varphi_k=0}^{\pi} \sin^{n-1-k} \varphi_k d\varphi_k. \end{aligned} \quad (8)$$

Calculating the phase average value over the surface of unit sphere, for the first amplitude

$$|b_1|^2 = \frac{1}{s} \int \left(|x_1|^2 + |x_2|^2 \right) ds =$$

$$= \int_0^{2\pi} \sin^2 \theta_1 d\theta_1 \int_0^\pi \prod_{k=1}^{40} \sin^{k+2} \theta_{k+1} d\theta_{k+1} = 0,047619, \quad (9)$$

where the integral is calculated by using the Nest function in Mathematica Wolfram¹. We found that the phase average value for the first amplitude equals (0,047619), whilst the time average over a representative set of initial data (100 points) is (0.0519488).

It is found that they are close to each other. The relative error $\Delta = \frac{|\bar{a}_s|^2 - |b_1|^2}{|b_1|^2} = 9\%$.

In the case of increasing the number of subharmonics, the accuracy of the solution will be increased.

Table 1

A comparison between the time average and the phase averaged is formulated for five subharmonics

S	$ a_s 2, a(t=0) = x^0$	$\frac{1}{100} \sum_{i=1}^{100} \bar{a}_s^i ^2, a_s^i(t=0) = x_i^0$	Δ
$s = 1$	0,0529904	0,0519488	9%
$s = 2$	0,0423804	0,0436673	8%
$s = 3$	0,0510327	0,0505745	6,2%
$s = 4$	0,049629	0,0505255	5,1%
$s = 5$	0,046456	0,0461685	3%

3. Numerical results

Reynolds stresses are almost always normalized by internal scales, viscous wall distance – y^+ and wall friction velocity – u_τ .

However, the behavior of the square longitudinal pulsation and its root-mean-square value are studied here when they are normalized to the average velocity $U(y^+)$. Figure 2 shows the behavior of the square longitudinal pulsation (7) related to the average velocity in a turbulent boundary layer for four different Reynolds numbers from $Re_\theta = 900$ to $Re_\theta = 1800$. Numerical results of this work show that the position of the maximum value increases with the growth of Reynolds number. Relation u^2 with respect to the average flow velocity, near the wall, at different values of Reynolds number, tends to zero as shown in Figure 2. The limit value of $\sqrt{u^2}/U$ at the wall is approximately ranged from 0,38 to 0,43, over range of Reynolds number ($900 < Re_\theta < 1800$). As shown in Fig. 3, a good agreement with numerical [11] and experimental results [12], is noticed. Figure 4 shows the root mean square pulsations of the longitudinal velocity $u_{rms}^+ = \sqrt{(u^+)^2}$ in a turbulent boundary layer at $Re_\theta = 1840$.

¹ Mathematica Wolfram Research 5.0

Fig. 2. Behaviour of turbulence intensity $\overline{(u(y^+))^2} / \overline{(U(y^+))^2}$ at $Re_\theta = 900 - 1800$, $U(y^+)$ – the profile of the average longitudinal velocity in the turbulent boundary layer.

Fig. 3. Distribution of turbulence intensity along the flow when scaling to the average velocity, at $Re_\theta = 900 - 1800$.

Fig. 4. u_{rms}^+ is a function of y^+ , at $Re_\theta = 1840$. Solid line (—) – present study ; (“■”) – DNS [13]; (“●”) – [14]; Experimental results (“◆”) – [15], (“▼”) – [16].

Fig. 5. Distribution of the Reynolds shear stress at different values of the Reynolds number $Re_\theta = 900 - 1800$

It is found that results obtained using the waveguide model are quantitatively and qualitatively consistent with numerical [13, 14] and experimental [15, 16] results. In terms of the waveguide model [17], a good comparison with Klebanov's experimental data [18] is deduced, that investigated the behavior of the turbulent shear stress as a function of the normal y coordinate. So that, the behavior of the Reynolds stresses related to the average velocity $U(y^+)$ is studied at different values of the Reynolds number in Fig.5. It is noticed that Reynolds number has weak effect in the viscous layer, but after that the maximum value are increased with increasing Reynolds number.

4. Conclusion

In this paper, the behavior of turbulence intensity and shear stress at different Reynolds numbers is investigated. The study is carried out for the incompressible fluid flow in the case of a zero longitudinal pressure gradient. The average values can be determined by averaging over space (the surface of a unit sphere), due to the ergodic behavior of the dynamic system. Turbulence statistics are normalized by the average velocity U . The results obtained for the coherent component using a waveguide model are quantitatively and qualitatively consistent with numerical and experimental results.

Статья поступила в редакцию 20.10.2021 г.

REFERENCES

1. Zharov, V.A. Model representation of a coherent structure in a developed turbulent boundary layer // Scientific Notes of TsAGI. 2014. v. XLV, issue №5.
2. Kadomtsev B.B. Plasma turbulence. In the book: Questions of plasma theory, issue 4. – M.: Atomizdat. – 1964. – pp. 188–339.
3. Landahl M.T. A wave-guide model for turbulent shear flow // J. Fluid Mech. – 1967. – V. 29, pt. 3. – P. 441 – 459.
4. Davidson R.C. Methods in nonlinear plasma theory. N.Y.; L.: Acad. Press, 1972. – 356 p. (Pure and applied physics; V. 37.)
5. Hussain A.K.M.F. Coherent structures – reality and myth // Phys. Fluids. –1983. – V. 26. N 10. – P. 2816 – 2863.
6. Zharov, V.A., Lipatov I.I., Selim, R.S. Spectral characteristics of incompressible fluid flow in a turbulent boundary layer// Bulletin of the Moscow State Regional University. Series: Physics-Mathematics. 2020. No. 4. pp. 12-27.
7. Колмогоров А. Н. Избранные труды. Математика и механика. М: Наука, 1985. 470 с.
8. Musker, A.J. Explicit expression for the smooth wall velocity distribution in turbulent boundary layer // AIAA Journal. 1979, V. 17 (6), p. 655-657.
9. Selim, R. S. Eigenmodes of the Orr-Sommerfeld equation in a developed turbulent boundary layer // Proceedings of May 2019. No. 109. DOI:10.34759/trd-2019-109-5.
10. Madelung. E. Mathematical apparatus of physics. Reference manual. M.: Fizmatlit, 1961 – 618 s.
11. Antonia, R. A., Kim, J. Low Reynolds number effects on near-wall turbulence // J. Fluid Mech. 1994. v. 276, 61–90.
12. Alfredsson. P. J., Johansson, A. V., Haritonidis, J. H. and Eckelmann, H. The fluctuation B wall-shear stress and the velocity field in the viscous sublayer // Phys. Fluids. 1988. v. 31, 1026–1033.

13. Dong, L, Kun, L, and Jianren, F. Direct numerical simulation of heat transfer in a spatially developing turbulent boundary layer // *Physics of Fluids*. 2016. v. 28, 105104.
14. Wu, X. and Moin P, Transitional and turbulent boundary layer with heat transfer // *Phys. Fluids*. 2010.v. 22, 085105
15. Purtell, L. P. Turbulent boundary layer at low Reynolds number // *Phys. Fluids*. 1981. v. 24, 802.
16. Karlsson, R. I. and. Johansson .T, "LDV measurements of higher order moments of velocity fluctuations in a turbulent boundary layer", in *Laser Anemometry in Fluid Mechanics*, edited by R. J. Adrian (Lisbou, Portugal, 1988), pp. 273–289.
17. Selim, R.S. Determination of Reynolds shear stress from the turbulent mean velocity profile and spectral characteristics of Orr-Sommerfeld -Squire equations // *Journal of Physics: Conference series*.2021. v. 2056 012017.
18. Klebanoff P S. Characteristics of turbulence in a boundary layer with zero pressure gradients. // *NACA*. 1955. Rep. 1247.

БИБЛИОГРАФИЧЕСКИЙ СПИСОК

1. Жаров, В.А. Модельное представление когерентной структуры в развитом турбулентном пограничном слое // *Учёные записки ЦАГИ*.2014.том XLV, вып №5.
2. Кадомцев Б.Б. Турбулентность плазмы. В кн.: *Вопросы теории плазмы*, вып. 4. – М.: Атомиздат. – 1964. –С. 188–339.
3. Landahl M.T. A wave-guide model for turbulent shear flow // *J. Fluid Mech.* – 1967. – V. 29, pt. 3. – P. 441 – 459.
4. Davidson R.C. *Methods in nonlinear plasma theory*. N.Y.; L.: Acad. Press, 1972. – 356p. (Pure and applied physics; V. 37.)
5. Hussain A.K.M.F. Coherent structures – reality and myth // *Phys. Fluids*. – 1983. – V. 26. N 10. – P. 2816 – 2863.
6. Жаров В.А., Липатов И.И., Селим Р.С. Спектральные характеристики течения несжимаемой жидкости в турбулентном пограничном слое// *Вестник Московского государственного областного университета. Серия: Физика-математика*. 2020. № 4. С. 12–27.
7. Kolmogorov A. N. *Selected works. Mathematics and mechanics*. M: Nauka, 1985. 470 p.
8. Musker, A J. Explicit expression for the smooth wall velocity distribution in turbulent boundary layer // *AIAA Journal*.1979, V.17 (6), p. 655–657.
9. Селим, Р. С. Собственные моды уравнения Орра-Зоммерфельда в развитом турбулентном пограничном слое // *труды Маи*.2019. № 109. DOI: 10.34759/trd-2019-109-5.
10. Маделунг. Э. Математический аппарат физики. Справочное руководство. М.: Физматлит, 1961 – 618 с
11. Antonia, R. A., Kim, J. Low Reynolds number effects on near-wall turbulence // *J. Fluid Mech*. 1994. v. 276, 61–90.
12. Alfredsson, P. J., Johansson, A. V., Haritonidis, J. H. and Eckelmann, H. The fluctuation B wall-shear stress and the velocity field in the viscous sublayer // *Phys. Fluids*. 1988. v. 31, 1026–1033.
13. Dong, L, Kun, L, and Jianren, F. Direct numerical simulation of heat transfer in a spatially developing turbulent boundary layer // *Physics of Fluids*. 2016. v. 28, 105104.
14. Wu, X. and Moin P, Transitional and turbulent boundary layer with heat transfer // *Phys. Fluids*. 2010.v. 22, 085105.
15. Purtell, L. P. Turbulent boundary layer at low Reynolds number // *Phys. Fluids*. 1981. v. 24, 802.

16. Karlsson. R. I. Johansson T., "LDV measurements of higher order moments of velocity fluctuations in a turbulent boundary layer," in *Laser Anemometry in Fluid Mechanics*, edited by R. J. Adrian (Lisbou, Portugal, 1988), pp. 273–289.
17. Selim. R.S. Determination of Reynolds shear stress from the turbulent mean velocity profile and spectral characteristics of Orr-Sommerfeld -Squire equations // *Journal of Physics: Conference series*. 2021. v. 2056 012017.
18. Klebanoff P S. Characteristics of turbulence in a boundary layer with zero pressure gradients. //NACA. 1955. Rep. 1247.

ИНФОРМАЦИЯ ОБ АВТОРЕ

Селим Рами Салах Сабер – аспирант кафедры компьютерного моделирования Московского физико-технического института (национального исследовательского университета); Ассистент преподавателя на факультете Естественных наук Университета г. Танта.

e-mail: selim.rs@phystech.edu.

INFORMATION ABOUT THE AUTHOR

Ramy Salah Saber Selim – Postgraduate Student, Department of computer Modeling, Moscow Institute of Physics and Technology (National Research University); Teaching assistant at the Faculty of Natural Sciences of the University of Tanta.

e-mail: selim.rs@phystech.edu

ПРАВИЛЬНАЯ ССЫЛКА НА СТАТЬЮ

Селим Р. С. Турбулентная статистика с точки зрения когерентной структуры в пограничном слое // Вестник Московского государственного областного университета. Серия: Физика-Математика. 2021. № 4. С. 75–85.

DOI: 10.18384/2310-7251-2021-4-75-85

FOR CITATION

Selim R. S. Turbulent statistics in terms of coherent structure in the boundary layer In: *Bulletin of the Moscow Region State University. Series: Physics-Mathematics*, 2021, no. 4, pp. 75–85.

DOI: 10.18384/2310-7251-2021-4-75-85