ФИЗИКА

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EMERGENT GRAVITY, ENTROPY, AND QUANTUM ENTANGLEMENT

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Abstract. A brief review of a set of ideas which relate gravity, entropy and quantum entanglement is given. The key idea is that gravity is an emergent or induced, in Sakharov's sense, phenomenon and it is holographic, as has been suggested by t'Hooft. The low-energy physics is described by effective equations of some mi- croscopical degrees of freedom. It is important that the entanglement entropy of these microscopic constituents across a surface is given by the Bekenstein-Hawking entropy formula. It means that all microscopic details are encoded in macroscopic low-energy constants. It is very similar to what happens in thermodynamics so that gravity, by following the Verlinde suggestion, may be interpreted as an entropic gravity. An explicit realization of this idea appears in a holographic description of entanglement entropy in conformal field theories.
Keywords: entropy of quantum entanglement, induced gravity, quantum gravity, quantum theories which allow a dual description in terms of gravity

1. Introduction

A fundamental problem of the modern cosmology is related to the origin and nature of the dark energy. The current observational data indicate that the dark energy may be in a form of cosmological constant and, thus, the resolution of the problem requires understanding properties of the physical vacuum. Due to quantum effects the vacuum can be viewed as a sort of media which possesses energy density, pressure, polarization and etc. In a conventional QFT the energy of vacuum fluctuations is not defined because of ultraviolet divergences. The problem of the cosmological constant is that for different natural cutoff parameters (such as electroweak, SUSY, or Planck scales) the vacuum energy is many orders of magnitude higher than the cosmological value. This fact may indicate that our present knowledge of fundamental physics is too incomplete.

The aim of this paper is to describe and analyse some ideas which enlarge our vision of quantum gravity. All these ideas are connected in a sense that low-energy gravity is considered as an emergent or induced gravity and a generalized notion of entropy takes place.

2. Induced gravity

The fact that gravity is an emergent phenomenon dates back to ideas of the last century suggested by A.D. Sakharov [1] who noticed that the leading part of the one-loop effective action on a curved manifold behaves as an Einstein action

$$\ln\det\left(-\nabla^2+m^2\right)\cong\frac{1}{16\pi G_{\text{eff}}}\int\sqrt{g}h^4x\left(2\Lambda_{\text{eff}}+R+a_{\text{eff}}R^2+\ldots\right) \tag{1}$$

with ’induced’ or ’effective’ Newton coupling G_{eff} and a cosmological constant Λ_{eff}

$$G_{\text{eff}}\sim M^2,\ \frac{G_{\text{eff}}}{\Lambda_{\text{eff}}}\sim M^2, \tag{2}$$

where M is a UV cutoff. The induced action (1) also includes higher curvature corrections. Constants G_{eff}, Λ_{eff} are analogous the Youngs modulus in the solid state physics, while gravitons are analogous to phonons. In the Sakharov theory the underlying degrees of freedom are just different species of relativistic fields.

In the string theory the same idea is realized in a more sophisticated scheme where low-energy gravity equations appear from tree-level amplitudes of closed strings. The similarity with Sakharov’s approach is that these diagrams can be reinterpreted as one- loop diagrams of open strings.

3. Black hole thermodynamics

The emergent nature of gravity is supported by properties of black holes. Dynamical laws of black holes can be interpreted as laws of thermodynamic systems. The mass of a black hole is identified with the energy of the system, the entropy of a black hole is given by the Bekenstein-Hawking entropy

$$S^{BH}=\frac{A(B)}{4G}, \tag{3}$$

where $A(B)$ is the area of the black hole horizon B. (Here and in what follows we assume that the Planck constant and the velocity of light are equal to unity). If we consider, for example, a Schwarzschild black hole, it is just an empty space. The problem is what are the degrees of freedom which allow one to explain its entropy.

One of the challenging tasks in quantum gravity is to provide a statistical-mechanical explanation of the Bekenstein-Hawking entropy. A possible source of S^{BH} are quantum correlations of underlying microscopical degrees of freedom across the black hole horizon. In the next section we explain why the induced gravity may be a useful guiding idea [2, 3, 4] to understand horizon correlations.

There is an important support of the idea that gravity is an emergent phenomenon. In 1995 Jacobson [5] made a remarkable observation that the Einstein equations can be derived by applying the first law of black hole thermodynamics to local Rindler horizons. This means that general laws of thermodynamics are applicable to quantum gravity degrees of freedom. Like phonons, gravitons are not fundamental degrees of freedom. They should not be canonically quantized.

Another intriguing concept has been suggested by E. Verlinde [6] who argued that gradients of the entropy of fundamental quantum gravity degrees of freedom might determine the gradients of the gravitational field. We return to Verlinde’s idea latter.

4. Entanglement entropy

To understand horizon correlations we introduce the entropy of quantum entanglement.

Quantum entanglement is a property of quantum systems which is known from early days of quantum mechanics. Now this property is well understood, established and it is used in different research areas. An example of entangled states is the following system of two particles, each with its spins 'up' ($|\uparrow\rangle_i|$) or 'down' ($|\downarrow\rangle_i$):

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle_1|\uparrow\rangle_2 + |\uparrow\rangle_1|\downarrow\rangle_2) . \qquad (4)$$

If the state of the 1st particle cannot be measured, state of the second particle will be the mixed one, described by the density matrix

$$\rho_2 = \mathrm{Tr}_1 |\psi\rangle\langle\psi| , \qquad (5)$$

The trace here is taken over the states of the 1st particle. The degree of entanglement of two particles is quantified by an entropy

$$S_2 = -\mathrm{Tr}_2 \rho_2 \ln \rho_2 . \qquad (6)$$

This quantity is also called the entanglement entropy. A simple computation shows that $S_2 = 2 \ln 2$.

One can generalize this definition to the case of a system of many particles divided onto two or more subsystems. One can show that the entanglement entropy is always zero if the subsystems are disentangled. In manybody systems, including quantum field theories, entanglement entropy $S(B)$ can be introduced for subsystems which are spatially separated by a surface B. Explicit computations show that this entanglement entropy is divergent and it behaves in the leading order as

$$S(B) = \frac{A(B)}{a^{D-2}} , \qquad (7)$$

where $A(B)$ is the area of B, a is a cutoff parameter with the dimensionality of a length, D is the number of space-time dimensions. If B is the black hole horizon and a is of the order of the Planck length the entropy (7) has the same order of magnitude as the Bekenstein-Hawking entropy.

Thus, the entropy of a black hole may be the entropy of entanglement between quantum excitations which live near the horizon, inside and outside [7-9]. These degrees of freedom are related to the physics at the Planck scale, they are not conventional low-energy fields.

To make this explanation complete one has to assume that low-energy gravity action is also induced by quantum effects of the Planck degrees of freedom so that, like in Sakharov's approach [2], the Newton coupling $G \sim a^2$ in $D = 4$. Explicit models of black hole entropy in induced gravity are discussed in [3, 4].

5. The world as a hologram

The idea that gravity might be a holographic theory first appeared in papers by t'Hooft [10] and Susskind [11]. Roughly speaking, it asserts that in the presence of quantum gravity effects all the information about physics inside a region can be described in terms of a theory (one-dimension lower) which is set on a boundary of the region.

Original arguments presented by t'Hooft are very simple. Suppose we want to understand a possible number of microstates located in a region of a volume

$V = L^3$. In classical theory this number is determined by the entropy of quanta. One can make the entropy larger, for example, by pumping an energy E and increasing the temperature T. Thus, one may expect that entropy scales as the volume V, $S \sim T^3L^3$. However, one can increase the energy only until the value $E_{max} = L/G$ when the gravitational radius EG remains smaller then the size L of the system. The limiting value for the entropy is $S_{max} \sim \left(L/\sqrt{G}\right)^{3/2}$. If $E > E_{max}$ the system collapses to form a black hole. The entropy of the system will be dominated by the entropy of a black hole $S = S^{BH} \sim L^2/G$. By taking into account quantum gravity degrees of freedom t'Hooft concludes that the entropy scales as the boundary of the region. The theory on the boundary whose degrees of freedom contribute to the entropy is called dual with respect to the theory in the physical volume (in the bulk).

The ideas about the holographic nature of quantum gravity are realized explicitly in what is known now as AdS/CFT correspondence. The conjecture made Maldacena and other authors [12-14] says that for gravity theories with a negative cosmological constant the dual theories are certain types conformal field theories (CFT's) one dimension lower than the theory in the bulk. (We do not provide here a precise formulation of the Maldacena conjecture. It appeals to the string theory.) Abbreviation 'AdS' means 'anti-de Sitter', since solutions in theories with a negative cosmological constant are asymptotically anti-de Sitter geometries.

6. Holographic entanglement entropy

For realistic condensed matter or field systems the entanglement entropy associated to spatial separation of the system is a non-trivial function of microscopical parameters. Its calculation is technically quite involved. Some progress in computations has been achieved either for one and two-dimensional spin chains or in case of non-interacting QFT's. Analytical or numerical computations of the entropy in the regime of strong couplings are not available.

In 2006 Ryu and Takayanagi [15, 16] by using AdS/CFT correspondence made a remarkable conjecture regarding entanglement entropy in conformal field theories. If a d-dimensional CFT admits a dual description in terms of an AdS gravity, the entanglement entropy associated with a partition of the CFT space by an entangling surface B is given by the Bekenstein-Hawking formula

$$S_d(B) = \frac{A(\tilde{B})}{4G_{d+1}} \ . \tag{8}$$

Here G_{d+1} is the higher dimensional Newton coupling and $A(\tilde{B})$ is the area of a co-dimension 2 hypersurface $\tilde{B}$ located in the bulk. $\tilde{B}$ is defined as minimal (least area) hypersurface with the condition that its asymptotic boundary is conformally equivalent to the physical entangling surface B.

All information about microscopical content of a given CFT is encoded in coupling constants of the bulk AdS graviy. Specification of a quantum state of the CFT is determined by the choice of the bulk solution. Ryu-Takayanagi formula (8) passes a number of non-trivial tests. It is important that (8) allows one to study entanglement entropy in strongly correlated systems.

7. Entanglement entropy in quantum gravity

Let us return now to the question about entanglement of quantum gravity degrees of freedom. In case of black holes, when the entangling surface is the black hole horizon, the entanglement entropy seems to be measured by the Bekenstein-Haking formula. What one can say about arbitrary entangling surfaces? By taking Ryu-Takayanagi formula (8) as a guide a number of arguments have been presented by the author of this paper [17] that the entanglement entropy of fundamental degrees of freedom lying in a constant time slice and spatially separated by a surface B is

$$S(B)=\frac{A(B)}{4G}. \tag{8}$$

Here G is the Newton coupling and A is the area of B. Equation (9) holds in the semiclassical approximation if the low-energy limit of the fundamental theory is the Einstein gravity.

As we pointed out earlier, for realistic condensed matter systems the entanglement entropy is a non-trivial function of microscopical parameters. The remarkable implication of (9) is that the entanglement entropy in quantum gravity may not depend on a microscopical content of the theory, it is determined solely in terms of geometrical characteristics of the surface and low-energy gravity couplings.

Another feature established in [17] is related to the shape of the separating surface. Because $S(B)$ includes contributions of all fundamental degrees of freedom quantum fluc- tuations of the geometry should be taken into account in a consistent way. For static space-times this requires that B is minimal surface, i.e. a surface with a least area.

Quite recently Maldacena and Lewkowycz [18] came to entropy formula (9) in an alternative way. They called such entropy generalized gravitational entropy.

Let us also note that results of [17] have an interesting relation to Verlinde's entropic gravity [6]. The hypothesis of [6] is based on a number of assumptions for so called 'holographic screens' which store an information about fundamental microstates ('bits') in such a way that a related entropy is proportional to the area of the screen. A variational formula for the entropy of the screen under the action of a point-like particle is postulated and plays a key role in the arguments. If the holographic screens are identified with minimal surfaces the variational formulas just follow from the properties of minimal surfaces, see [17].

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ПОРОЖДАЕМАЯ ГРАВИТАЦИЯ, ЭНТРОПИЯ И КВАНТОВОЕ ПЕРЕПУТЫВАНИЕ

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Аннотация. Дается краткий обзор научных идей, которые связывают такие понятия как гравитация, энтропия и квантовое перепутывание. Ключевая идея состоит в том, что гравитация может быть "порождаемой" или "индуцированной", в духе идей А.Д. Сахарова, а также гравитационные явления могут обладать голографическими свойствами, как впервые было отмечено Г. 'т Хоофтом. Низкоэнергетическая физика описывается эффективными уравнениями некоторых микроскопических степеней свободы. Важно, что энтропия перепутывания этих микроскопических конституентов на некоторой поверхности дается формулой Бекенштейна-Хокинга. Это означает, что все микроскопические детали теории оказываются "зашифрованными" в конечном наборе макроскопических параметров. Это настолько напоминает то, что происходит в обычной термодинамике, что сама гравитация, следуя Г. Верлинде, возможно, имеет энтропийную основу. Подтверждением этих гипотез и их конкретной реализацией выступает голографическое представление энтропии перепутывания в определенном классе конформных теорий.

Ключевые слова: Энтропия квантового перепутывания, индуцированная гравитация, квантовая гравитация, квантовые теории с дуальным описанием в гравитации.